



Chapter 17: Recursion

Sections 17.1}17.2

Textbooks: Y. Daniel Liang, Introduction to Programming with C++, 3rd Edition
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Outline

- Introduction
- Example: Factorials

Motivations

- *Recursion is a technique that leads to elegant solutions to problems that are difficult to program using simple loops.*
- *A recursive function is one that invokes itself.*
- Suppose you want to find all the files under a directory that contains a particular word. How do you solve this problem? There are several ways to solve this problem. An intuitive solution is to use recursion by searching the files in the subdirectories recursively.

Outline

- Introduction
- **Example: Factorials**

Factorials in Math

$0! = 1$	-
$1! = 1$	$1 \times 0!$
$2! = 2 \times 1$	$2 \times 1!$
$3! = 3 \times 2 \times 1$	$3 \times 2!$
$4! = 4 \times 3 \times 2 \times 1$	$4 \times 3!$
$5! = 5 \times 4 \times 3 \times 2 \times 1$	$5 \times 4!$
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$	$6 \times 5!$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	$7 \times 6!$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	$8 \times 7!$
$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	$9 \times 8!$
$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	$10 \times 9!$

Computing Factorial

$$n! = n \times (n - 1) \times (n - 2) \times \dots \times 2 \times 1$$

$$0! = 1;$$

$$n! = n \times (n - 1)!; n > 0$$

$$\text{factorial}(0) = 1;$$

$$\text{factorial}(n) = n * \text{factorial}(n-1);$$

[ComputeFactorial](#)

Run

ComputeFactorial.cpp

```
#include <iostream>
using namespace std;

// Return the factorial for a specified index
long long factorial(int n)
{
    if (n == 0) // Base case
        return 1;
    else
        return n * factorial(n - 1); // Recursive call
}

int main()
{
    // Prompt the user to enter an integer
    cout << "Please enter a non-negative integer: ";
    int n;
    cin >> n;

    // Display factorial
    cout << "Factorial of " << n << " is " << factorial(n);

    return 0;
}
```

Factorial of 4 is 24

Computing Factorial

factorial(4)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

$$\text{factorial}(4) = 4 * \text{factorial}(3)$$

$$\text{factorial}(0) = 1;$$

$$\text{factorial}(n) = n * \text{factorial}(n-1);$$

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2)\end{aligned}$$

Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1))\end{aligned}$$

Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0)))\end{aligned}$$

Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1))\end{aligned}$$

Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1)\end{aligned}$$

Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1) \\ &= 4 * 3 * 2\end{aligned}$$

Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1) \\ &= 4 * 3 * 2 \\ &= 4 * 6\end{aligned}$$

Computing Factorial

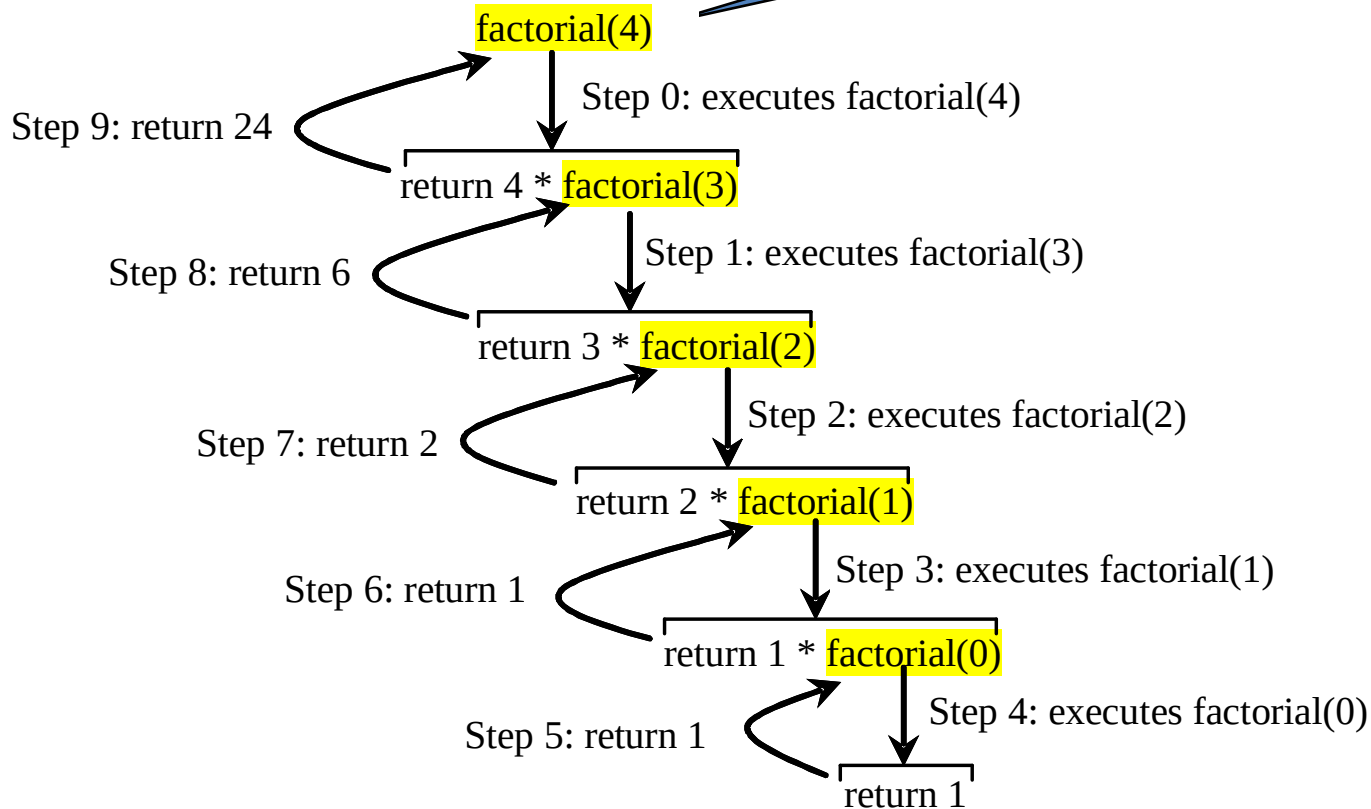
`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1) \\ &= 4 * 3 * 2 \\ &= 4 * 6 \\ &= 24\end{aligned}$$

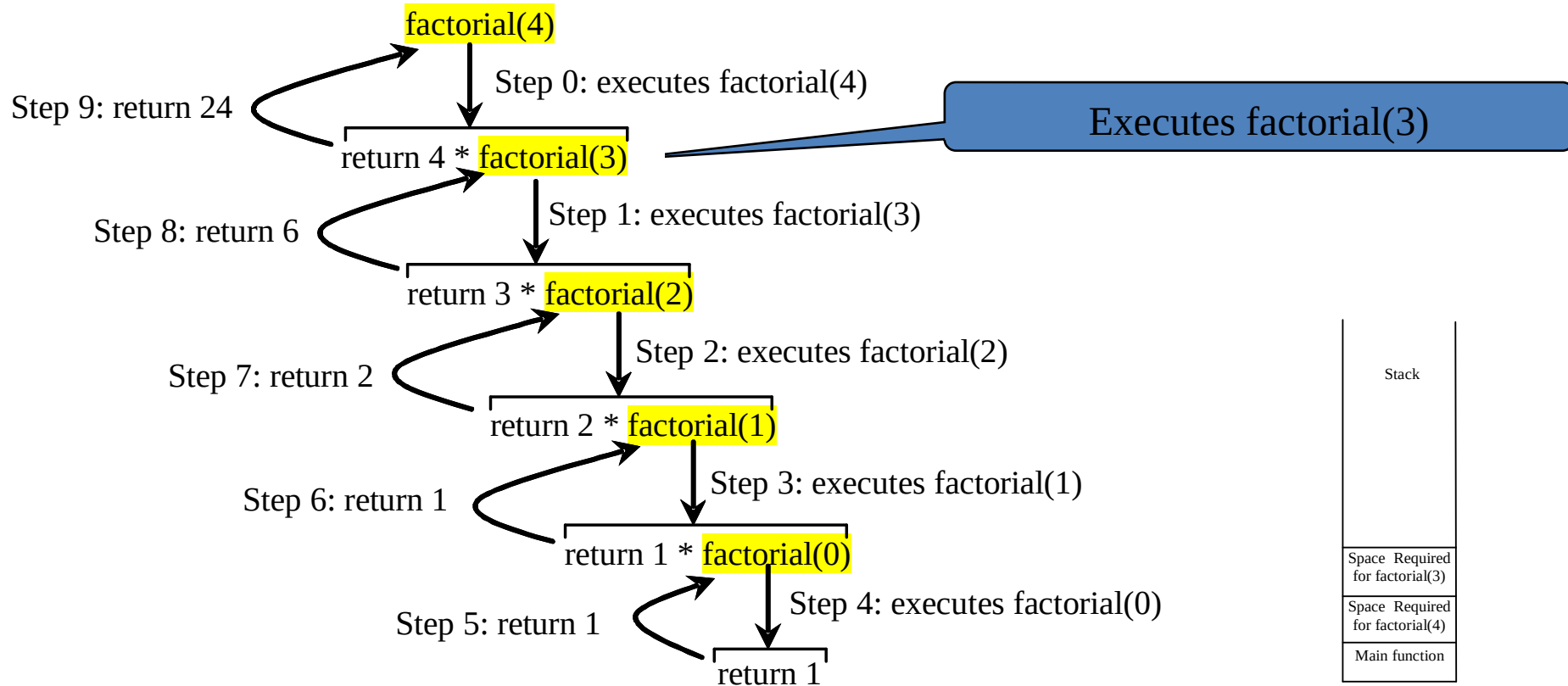
Trace Recursive factorial

Executes factorial(4)

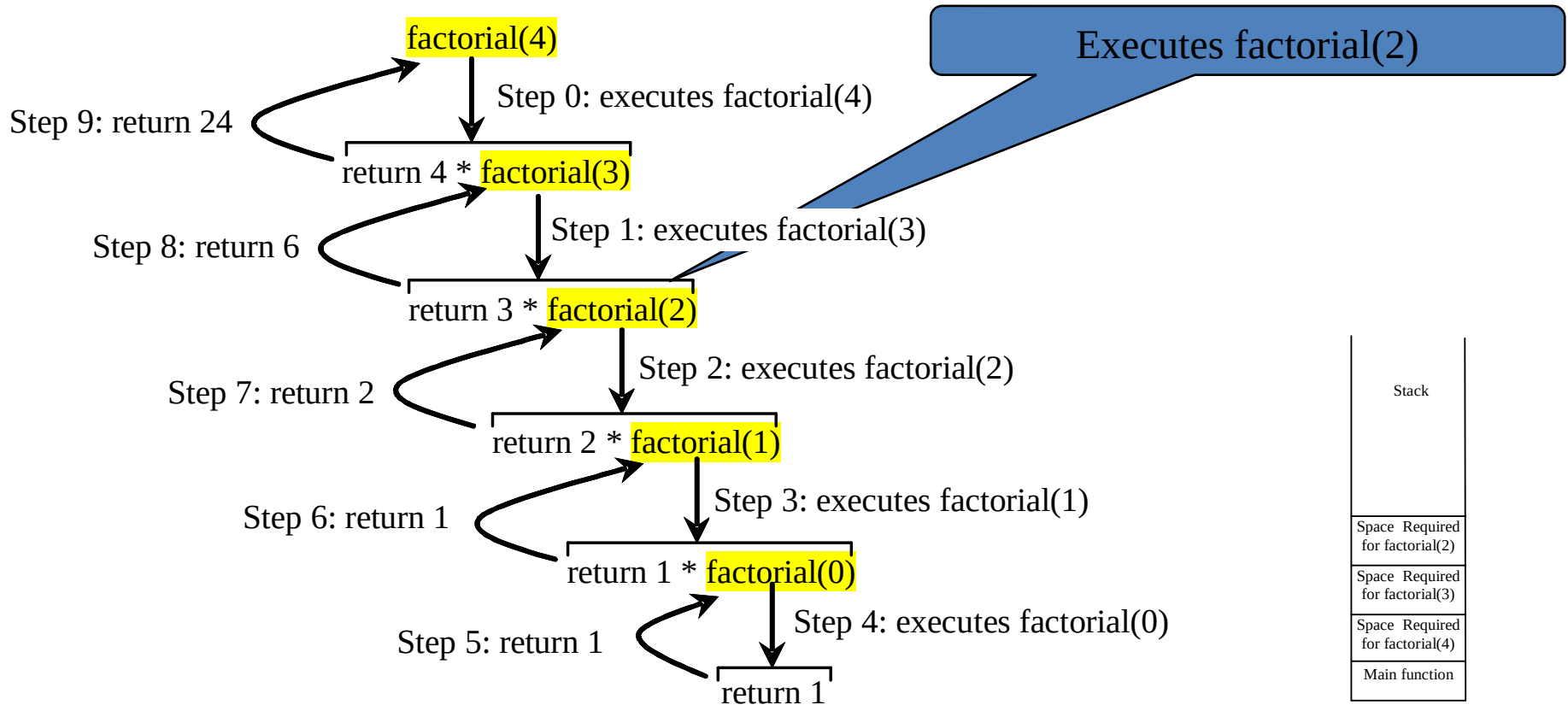


Stack
Space Required for factorial(4)
Main function

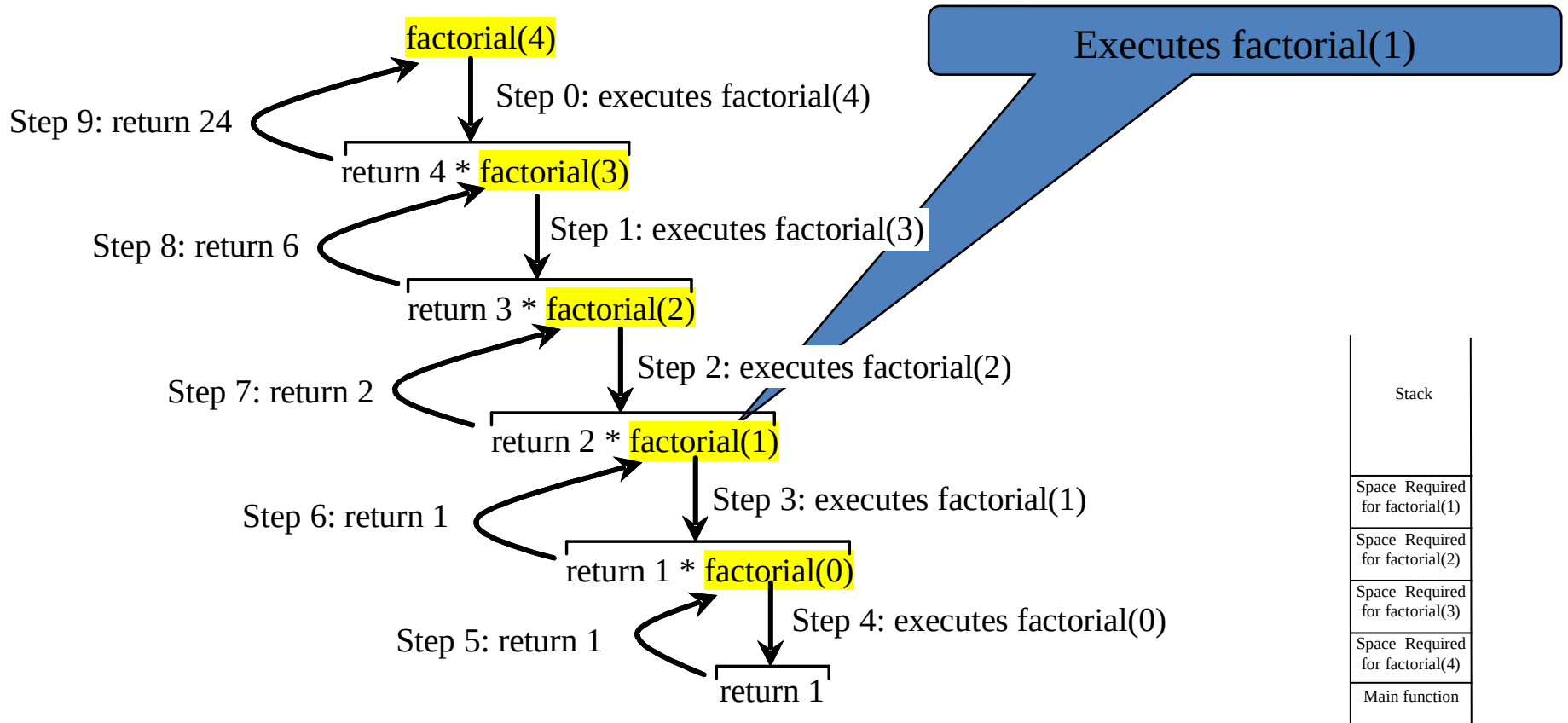
Trace Recursive factorial



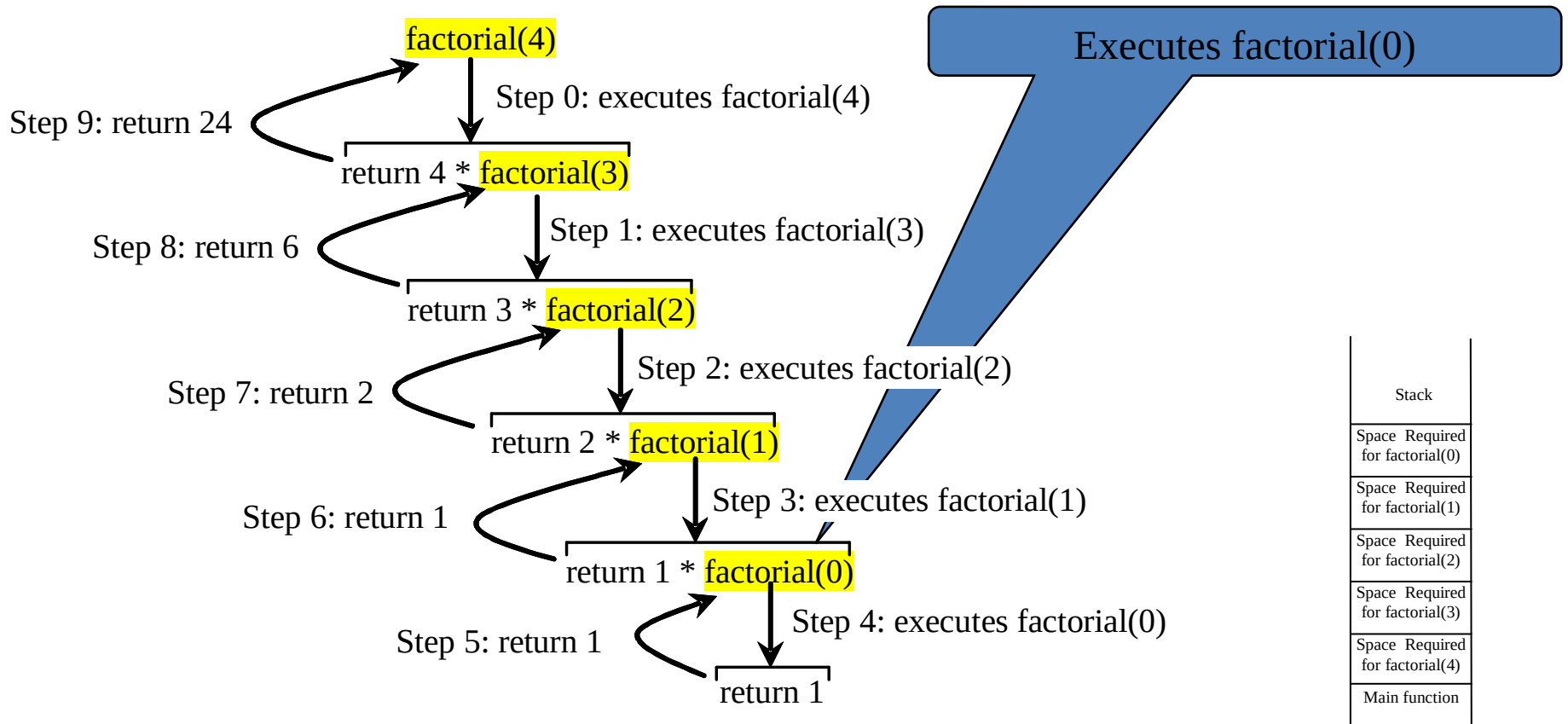
Trace Recursive factorial



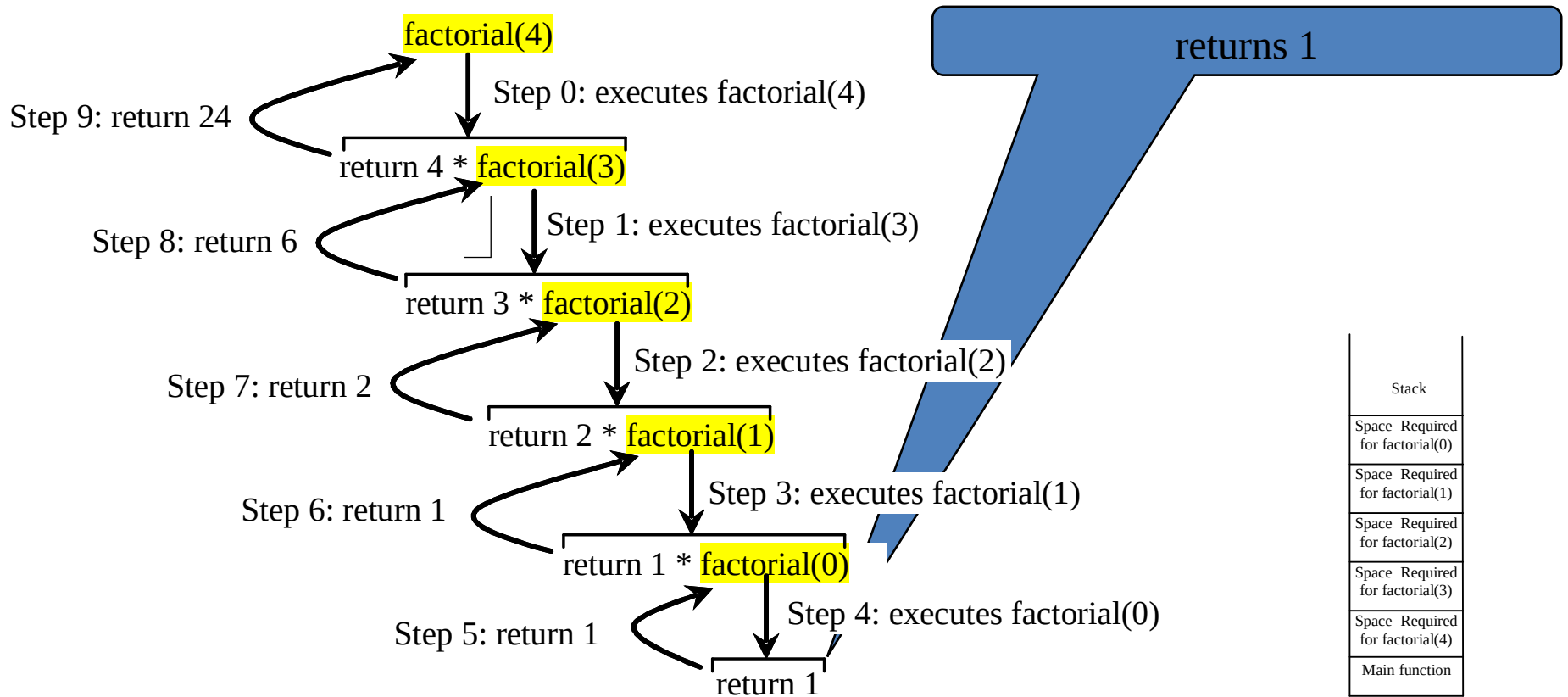
Trace Recursive factorial



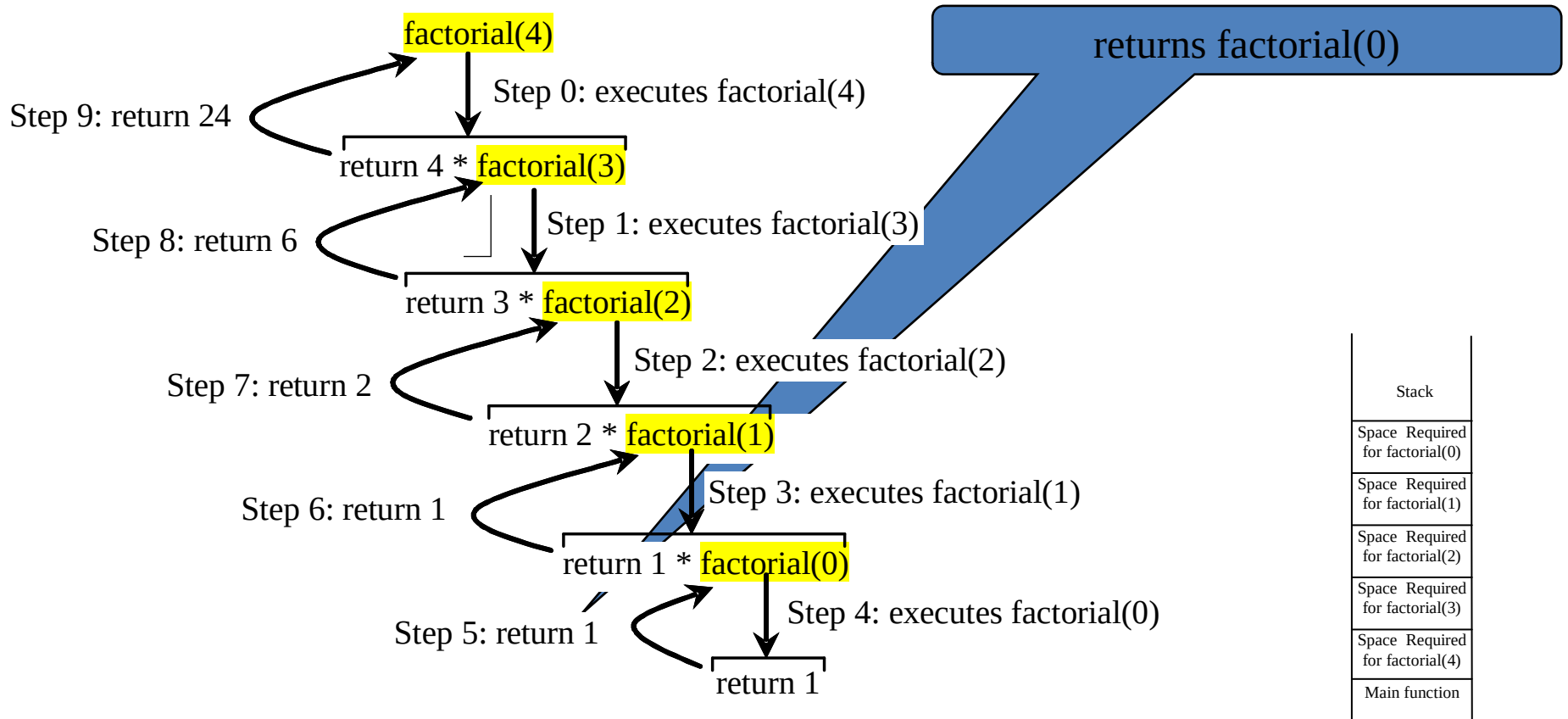
Trace Recursive factorial



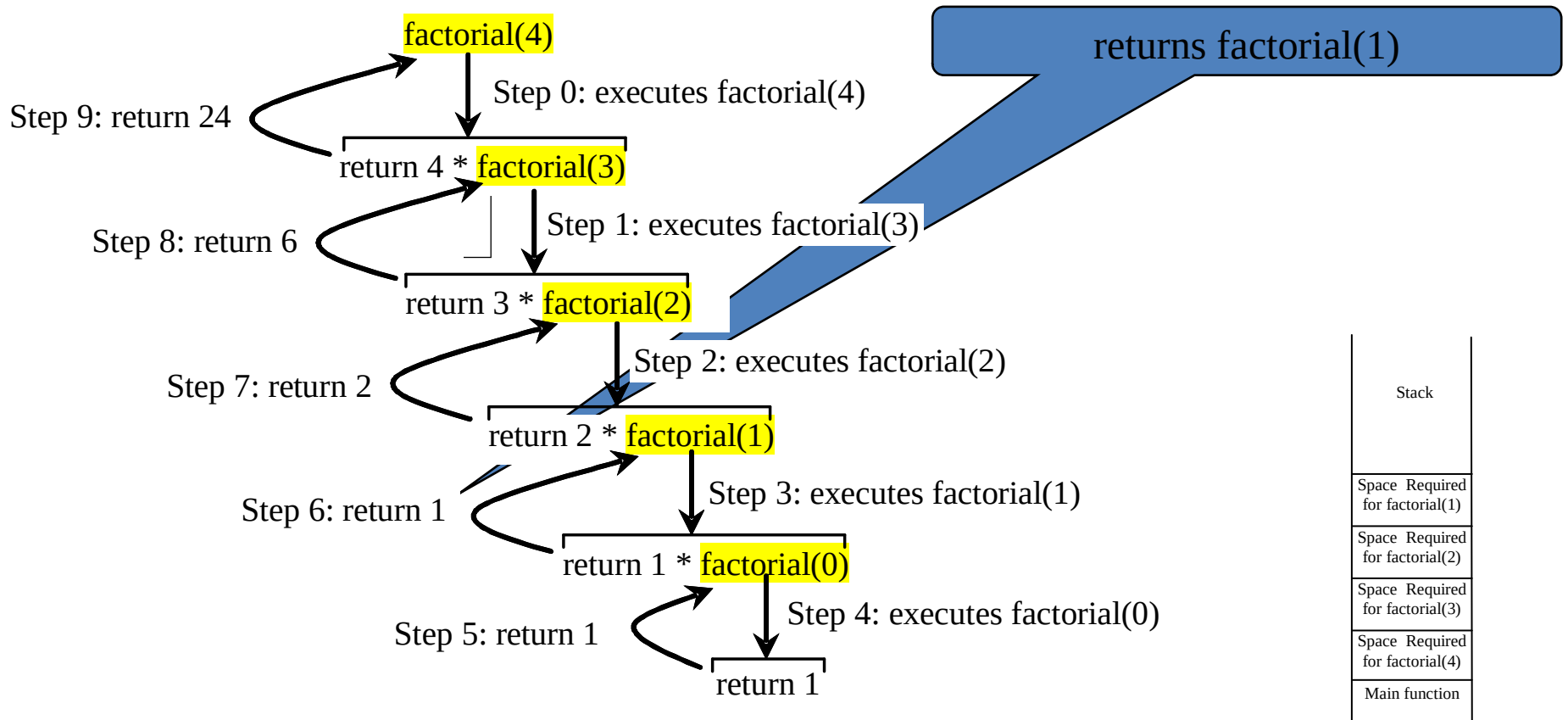
Trace Recursive factorial



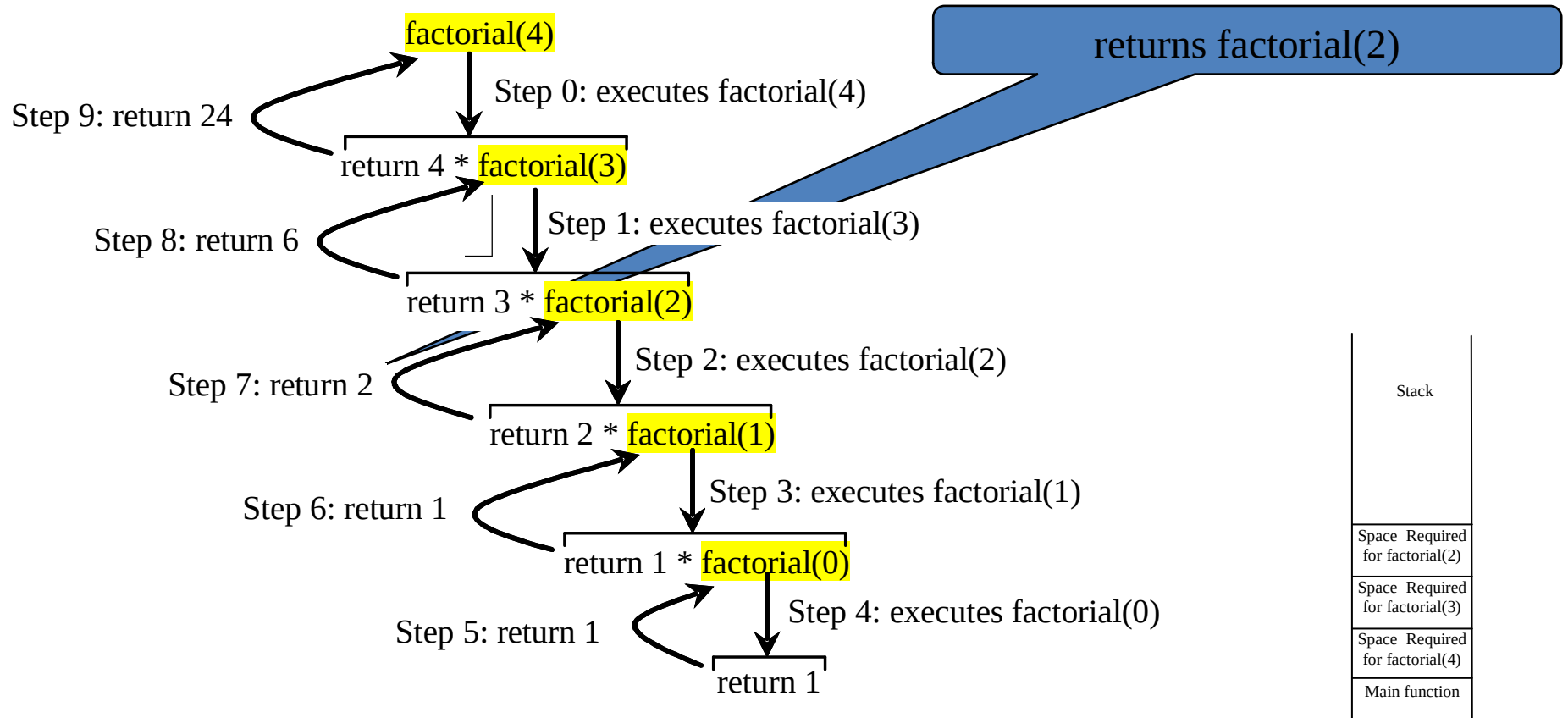
Trace Recursive factorial



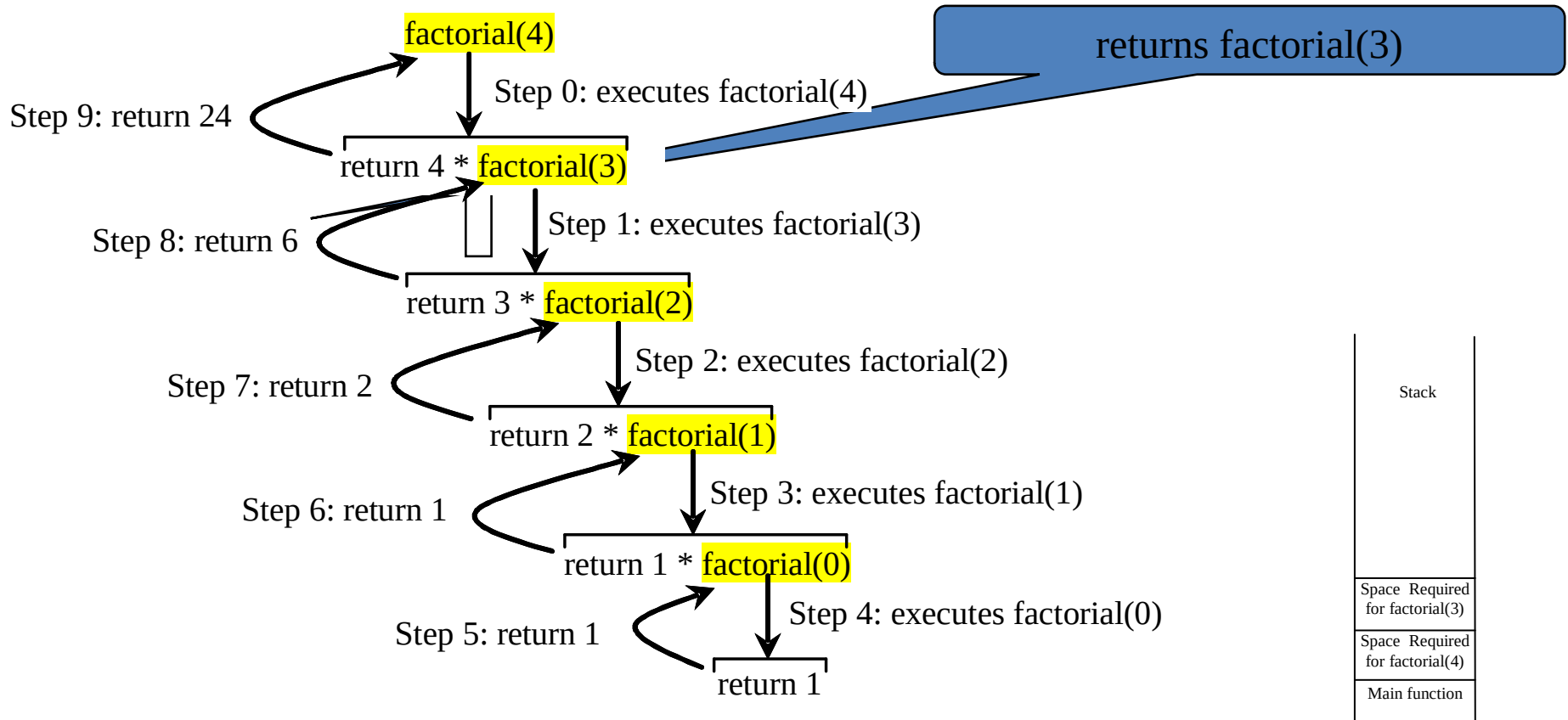
Trace Recursive factorial



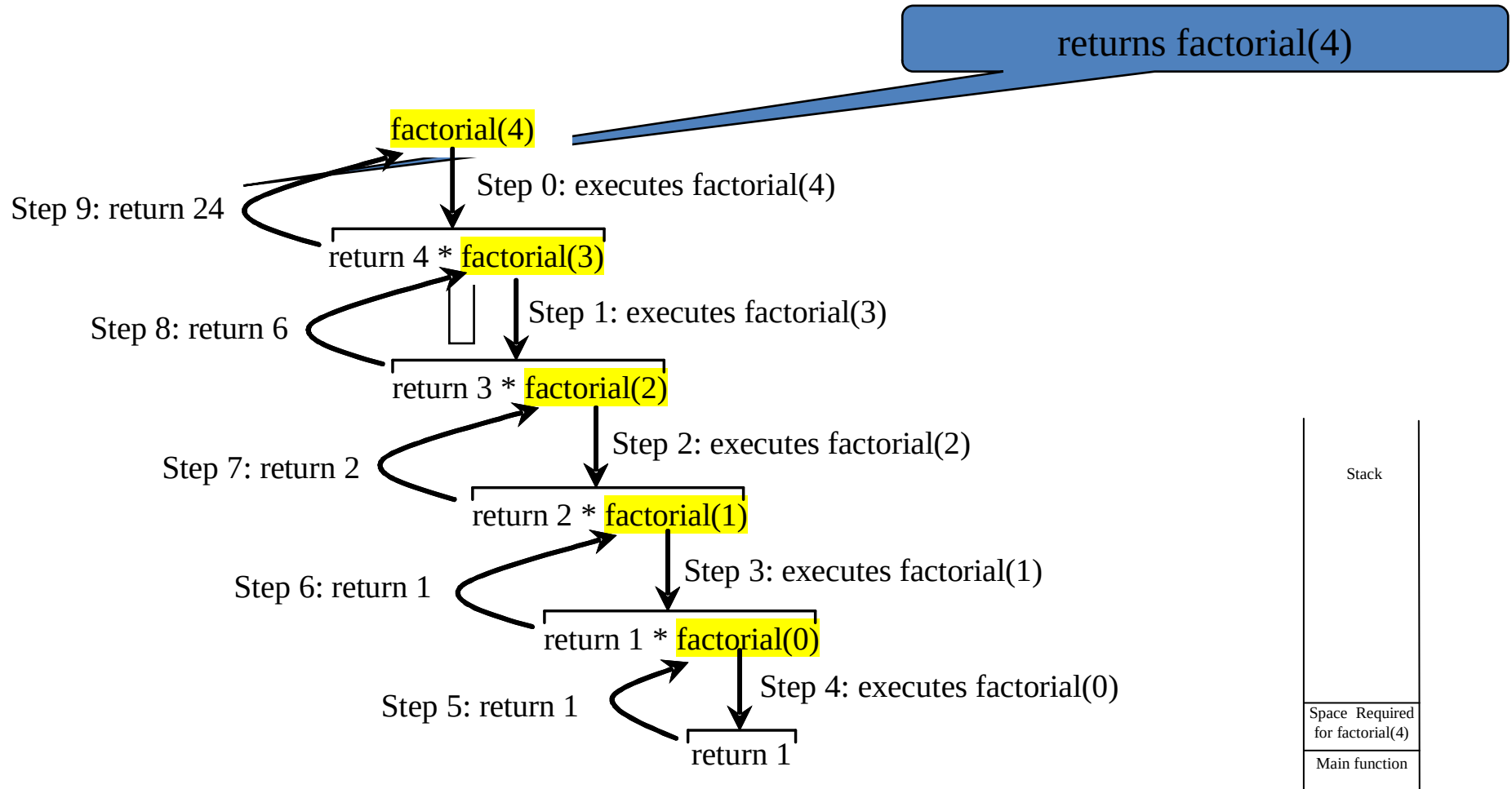
Trace Recursive factorial



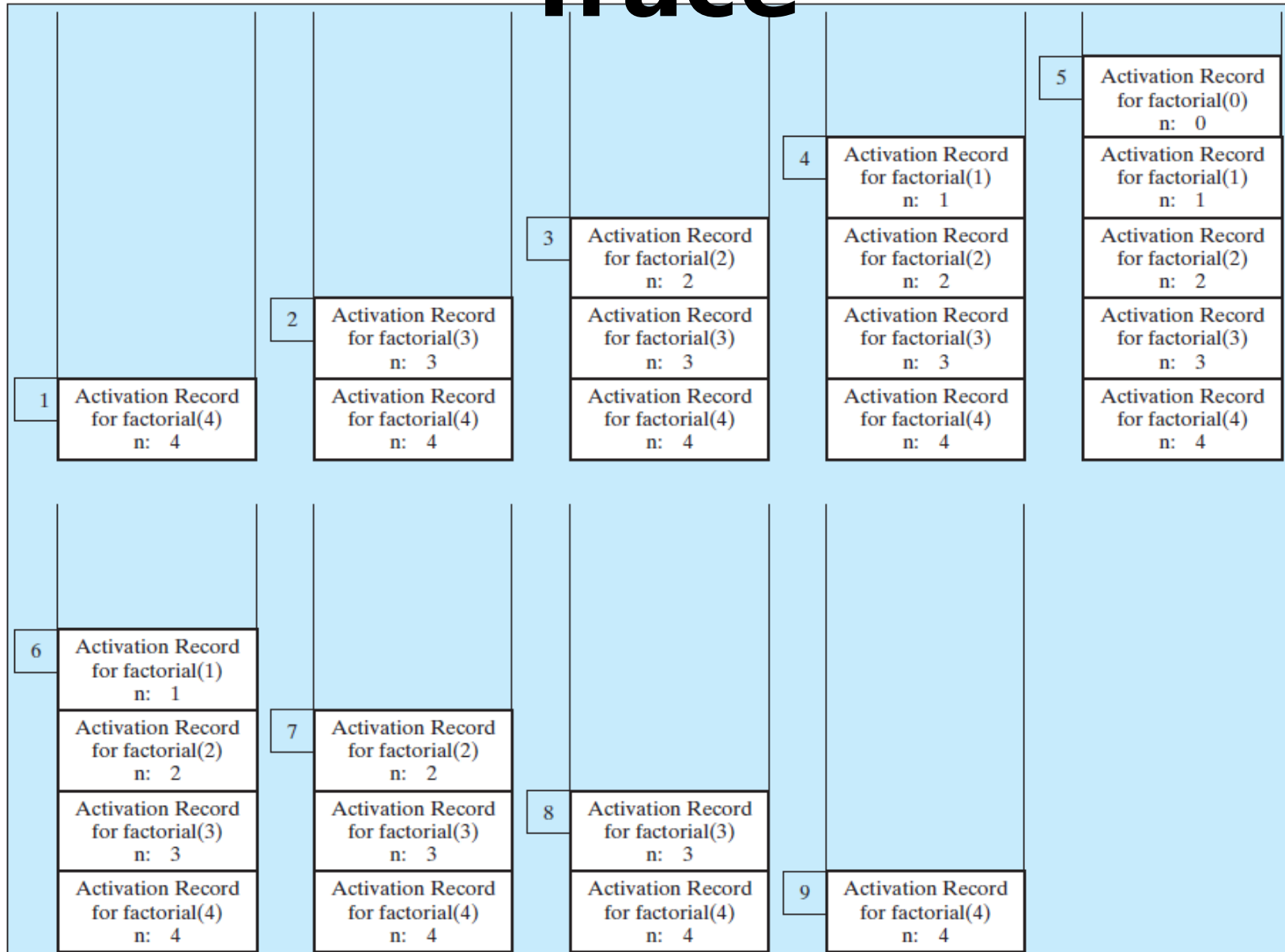
Trace Recursive factorial



Trace Recursive factorial



Factorial(4) Stack Trace



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- Introduction
- Example: Factorials